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Turbulence Model Behavior in Low Reynolds Number Regions of Aerodynamic Flowfields

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The behaviors of the widely used Spalart–Allmaras and Menter shear-stress transport turbulence models at low Reynolds numbers and under conditions conducive to relaminarization are documented. The flows used in the investigation include 2-D zero-pressure-gradient flow over a flat plate from subsonic to hypersonic Mach numbers, 2-D airfoil flow from subsonic to supersonic Mach numbers, 2-D subsonic sink flow, and 3-D subsonic flow over an infinite swept wing (particularly its leading-edge region). Both models exhibit a range over which they behave "transitionally" even with inflow values set to cause immediate growth of the turbulence quantities, in the sense that the flow is neither laminar nor fully turbulent, but these behaviors are different: the shear-stress transport model typically has a well-defined transition location, whereas the Spalart–Allmaras model does not. Both models are predisposed to delayed activation of turbulence with increasing freestream Mach number. Also, both models can be made to achieve earlier activation of turbulence by increasing their freestream levels, but too high a level can disturb the turbulent solution behavior. The technique of maintaining freestream levels of turbulence without decay in the shear-stress transport model, introduced elsewhere, is shown here to be useful in reducing grid dependence of the model's transitional behavior. Both models are demonstrated to be incapable of predicting relaminarization; eddy viscosities remain weakly turbulent in accelerating or laterally strained boundary layers for which experiment and direct simulations indicate turbulence suppression. The main conclusion is that these models are intended for fully turbulent high Reynolds number computations, and using them for transitional (e.g., low Reynolds number) or relaminarizing flows is not appropriate. Competing models which fare better in these areas have not been identified.

I. Introduction

THE Spalart–Allmaras (SA) turbulence model [1] and the Menter k–ω shear-stress transport (SST) turbulence model [2] have been widely used and trusted models for Reynolds-averaged Navier–Stokes (RANS) computations of aerodynamic flows for well over a decade. Recently, Rumsey [3] showed that, under certain circumstances, both of these models could exhibit inconsistent numerically induced transition regions (near the stagnation region of airfoils) that vary with grid density. These inconsistencies were unrelated to problems earlier identified for certain forms of k–ε that could yield arbitrary converged solutions [4,5]. The problem with the SA model was easily solved by either using a freestream turbulence level higher than a particular threshold level or by making a simple change to one of the model constants. Spalart and Rumsey [6] subsequently determined the inconsistency in the SST model to be primarily due to the grid-dependent decay rate of freestream turbulence for two-equation models. An accurate rendition of the decay would require grid spacings much finer than needed by the flowfield, and therefore be wasteful. The authors also made general recommendations for effective inflow conditions for turbulence models and showed how the addition of source terms in two-equation models can sustain the freestream ambient turbulence levels. This elimination of freestream decay not only reduces the aforementioned problems associated with grid dependency, but it is also more representative of the physics of the boundary layer-scale turbulence in both wind tunnel and flight.

Having achieved the ability to compute grid-consistent solutions, we now turn our attention to documenting the effects of Reynolds number and Mach number on the flowfields produced by these models. In particular, we focus on their inherent transitional behavior. It is important to recognize that, even when run in "fully turbulent" mode, turbulence models do not necessarily yield a fully turbulent solution everywhere in the boundary layer. There is often a region near the leading edge of aerodynamic bodies where the flow is effectively laminar because the eddy viscosity produced by the turbulence model is much lower than the molecular viscosity. The low values of eddy viscosity are a consequence of the turbulence model not having sufficient turbulence-production strength from the mean shear flow; this capacity is a strong function of the Reynolds number. The importance of checking computed results for unintended laminar behavior is sometimes stressed [7], but in reality it is probably not done very often. When checking, the flow at a location is often considered turbulent when $\mu_t/\mu_\infty > 1$ in the boundary layer above the body surface. Another criterion is the turbulence index i_t from Spalart and Allmaras [1], which has a value close to zero in a laminar region and close to 1 in a turbulent region.

It is important to note that the SA and SST turbulence models, along with many other models in wide use today, were not designed to predict transition. They do not include any transition modeling capability or "tuning" per se. Thus, any transitional behavior exhibited by the models should not be expected to agree with real physical transition processes. To begin with, they have no channel to input relevant information such as noise, surface roughness/waviness, frequency content of the ambient turbulence, and so on. The crucial sensitivity to pressure gradients and crossflow is also essentially absent. This is one reason why Spalart and Rumsey made recommendations for freestream levels that were not based upon matching freestream turbulence (Tu) levels from wind tunnel or

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flight, but rather upon considerations related to preserving potential cores in small geometry features and maintaining the integrity of the turbulence quantities throughout the boundary layers. Nonetheless, there is some qualitative correspondence in the trends between the models and experiment: at lower Reynolds numbers the computed laminar regions can be quite extensive, while at higher Reynolds numbers the boundary layers go turbulent earlier. Users should be made aware that laminar flow regions may be occurring in their computations, especially at low or moderate Reynolds numbers. Very large regions of laminar flow may signify that the turbulence models are being used outside of their intended range of applicability. In other words, these turbulence models were intended for use in predicting turbulent flows; if the Reynolds number is so low that the flowfield is mostly laminar or transitional, then use of a transition model [8–14] would be more appropriate.

Because low Reynolds number flows are common for a wide variety of applications, including micro-air vehicles and many wind-tunnel experiments, awareness of how the models behave in these circumstances, and guidance on whether they should even be trusted at all, can be critical. This paper seeks to document some of these characteristic behaviors of the SA and SST turbulence models. The goal is not to advocate using these models to predict transition (they should not), but rather to demonstrate the kinds of transitional behaviors that can occur when using them in supposedly fully turbulent simulations. In other words, we demonstrate how these models behave for low Reynolds number flowfields when fully turbulent computations are specified/desired. To our knowledge this type of study has not been done before. It is hoped that this documentation will provide useful guidance for users *before* using the SA or SST models for any particular application at low or moderate Reynolds number. We also investigate their ability to relaminarize in strongly accelerating boundary-layer flows. Revealing applications are given for a flat plate and the NACA 0012 airfoil in both subsonic and supersonic flow conditions, and also for 2-D sink flow and 3-D two-element infinite swept-wing computations in subsonic conditions.

II. Numerical Method

The computer code CFL3D [15] solves the three-dimensional, time-dependent compressible RANS equations with an upwind finite volume formulation (it can also be exercised in a two-dimensional mode of operation for 2-D cases). Upwind-biased third-order spatial differencing is used for the inviscid terms, and viscous terms are centrally differenced. The code originally solved the thin-layer form of the equations (in each coordinate direction), but the full Navier–Stokes terms (i.e., cross-derivative terms) have recently been added. All solutions shown below use the full Navier–Stokes terms.

The CFL3D code is advanced in time with an implicit approximate factorization method. The implicit derivatives are written as spatially first-order accurate, which results in block tridiagonal inversions for each sweep. However, for solutions that use Roe flux-difference splitting [16], the block tridiagonal inversions are further simplified using a diagonal algorithm with a spectral radius scaling of the viscous terms.

The turbulence models, including SA and SST, are solved uncoupled from the mean flow equations using implicit approximate factorization. Their advective terms can be solved using either first-order or second-order upwind differencing, with first-order the default for the code.

The computer code FUN3D [17,18] was also employed for a few of the cases presented in this paper (as noted in the text), to demonstrate that different codes produce similar results in terms of transitionlike behavior for SA and SST. FUN3D is an unstructured-grid finite volume RANS solver in which the flow variables are stored at the vertices of the mesh. At interfaces delimiting neighboring control volumes, the inviscid fluxes are computed using an approximate Riemann solver. The convective flux scheme is Roe flux-difference splitting. Backward Euler time differencing is employed, and at each time step, the linear system of equations is approximately solved with a multicolor point-implicit procedure. The turbulence models are uncoupled from the mean flow equations.

III. Turbulence Models

The one-equation SA model is written in terms of the turbulence quantity $\tilde{\nu}$:

$$\frac{\partial \tilde{\nu}}{\partial t} + u_j \frac{\partial \tilde{\nu}}{\partial x_j} = C_{b1}(1 - f_{t2})\tilde{S}\,\tilde{\nu} - \left[C_{w1} f_w - \frac{C_{b1}}{\kappa^2} f_{t2}\right]\left(\frac{\tilde{\nu}}{d}\right)^2 + \frac{1}{\sigma}\left[\frac{\partial}{\partial x_j}\left((\nu + \tilde{\nu})\frac{\partial \tilde{\nu}}{\partial x_j}\right) + C_{b2}\frac{\partial \tilde{\nu}}{\partial x_i}\frac{\partial \tilde{\nu}}{\partial x_i}\right] \tag{1}$$

where a description of each of the terms is not given here, but can be found in the original reference [1]. The quantity $\tilde{\nu}$ is related to the eddy viscosity by

$$\mu_t = \rho \tilde{\nu} \frac{(\tilde{\nu}/\nu)^3}{(\tilde{\nu}/\nu)^3 + c_{v1}^3} \tag{2}$$

where ν is the molecular kinematic viscosity and $c_{v1} = 7.1$.

The two-equation SST model is written in terms of the two turbulence quantities k and ω. When including the additional sustaining terms proposed by Spalart and Rumsey [6], the form is

$$\frac{\mathrm{D}\rho k}{\mathrm{D}t} = \mathcal{P} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + \beta^* \rho \omega_{\mathrm{amb}} k_{\mathrm{amb}} \tag{3}$$

$$\frac{\mathrm{D}\rho\omega}{\mathrm{D}t} = \frac{\rho\gamma}{\mu_t}\mathcal{P} - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + 2(1 - F_1)\frac{\rho}{\sigma_{\omega 2}\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} + \beta\rho\omega_{\mathrm{amb}}^2 \tag{4}$$

with $\mathcal{P} = \tau_{ij}\partial u_i/\partial x_j \approx \mu_t \Omega^2$ and Ω is the vorticity magnitude. The eddy viscosity is given by

$$\mu_t = \rho \frac{a_1 k}{\max(a_1\omega, \Omega F_2)} \tag{5}$$

where $a_1 = 0.31$ and F_2 is a blending function. This model is identical to the original SST model in every respect except for the addition of constant sustaining terms $\beta^*\rho\omega_{\mathrm{amb}}k_{\mathrm{amb}}$ and $\beta\rho\omega_{\mathrm{amb}}^2$. In the freestream, these have the effect of canceling the destruction terms if $k = k_{\mathrm{amb}}$ and $\omega = \omega_{\mathrm{amb}}$. This paper [6] introduced the appellation "ambient values." Inside the boundary layer, the sustaining terms are generally orders of magnitude smaller than the destruction terms for reasonable freestream turbulence levels (say, Tu = 1% or less), and therefore have little effect. A complete description of each of the terms in the standard SST equations can be found in Menter [2].

IV. Results

In Spalart and Rumsey [6], the following recommendations were made for setting freestream turbulence levels. For the SA model: $\tilde{\nu}'_\infty = \tilde{\nu}_\infty/\nu_\infty = 3$. For the k–ω models: $k_\infty/u_\infty^2 = 1 \times 10^{-6}$ and $\omega_\infty L/u_\infty = 5$. Here, L is the scale of the aerodynamic device, usually the chord of the wing. In coming up with these recommendations, consideration was given to gaps in multi-element configurations as well as to both well-developed and leading-edge boundary layers. Also, these recommended values rely on the assumption that freestream decay is being prevented in the two-equation models (freestream decay does not occur for $\tilde{\nu}$ in the SA model). Table 1 shows the correspondence between the recommended values and other commonly referenced quantities and nondimensionalizations. Here, $\mathrm{Tu}(\%) = 100\sqrt{2/3(k_\infty/u_\infty^2)}$. For SA, $\mu_{t,\infty}/\mu_\infty = (\tilde{\nu}'_\infty)^4/[(\tilde{\nu}'_\infty)^3 + c_{v1}^3]$; for SST, $\mu_{t,\infty}/\mu_\infty = (k_\infty/u_\infty^2)/(\omega_\infty L/u_\infty)Re$. Note that for the SA model, the freestream nondimensional eddy viscosity is near 0.21, whereas for SST (or other k–ω models) it varies depending on Reynolds number. For

Table 1 Correspondence between turbulence variables in the freestream

Model	$\tilde{\nu}_\infty/\nu_\infty$	k_∞/u_∞^2	$\omega_\infty L/u_\infty$	Tu, %	$\mu_{t,\infty}/\mu_\infty$	k_∞/a_∞^2	$\omega_\infty\mu_\infty/(\rho_\infty a_\infty^2)$
SA	3	n/a	n/a	n/a	0.21044	n/a	n/a
SST	N/A	1×10^{-6}	5	0.08165	$(2\times10^{-7})Re$	$(1\times10^{-6})M^2$	$5M^2/Re$

example, for $Re = 100,000$: $\mu_{t,\infty}/\mu_\infty = 0.02$; for $Re = 1\times10^6$: $\mu_{t,\infty}/\mu_\infty = 0.2$; and for $Re = 1\times10^7$: $\mu_{t,\infty}/\mu_\infty = 2$.

A. Flat Plate

Two-dimensional zero-pressure-gradient flat plate computations were performed on a series of grids of size 273 × 193 (fine), 137 × 97 (medium), and 69 × 49 (coarse), with most of the runs on the medium grid. The grids extended over nondimensional distances $-0.33333 < x < 2$ and $0 < y < 1$. On the medium grid, the minimum grid spacing (wall normal direction) was 1×10^{-6}. This was fine enough to yield minimum y^+ levels well less than 0.1 at all conditions tested. The medium grid x-direction spacing was about 0.043, with clustering near the leading edge (x spacing of 0.002) at $x = 0$. There were 25 points upstream of the plate leading edge and 113 points on the plate itself. The three grids were of the same family, so the fine and coarse grids had double and half the medium grid spacing in both coordinate directions, respectively.

Boundary conditions were as follows. Symmetry conditions were imposed on the lower boundary faces located upstream of the leading edge at $x = 0$. On the plate, adiabatic solid wall conditions were imposed. The top boundary faces used far-field Riemann-type boundary conditions, and the downstream boundary used extrapolation. For subsonic flow, upstream faces used a characteristic method similar to the far-field Riemann method, except that total pressure and total temperature were set for the external state according to isentropic relations for the particular Mach number chosen. For supersonic flow, all inflow variables were specified. The far-field inflow turbulence variables were specified according to Table 1, and outflow turbulence variables were extrapolated from the interior. At the wall, $\tilde{\nu}_w = k_w = 0$ and ω_w was computed using the boundary condition from Menter [2].

The expected behavior for turbulence models for this type of flow is shown in Figs. 1a and 1b, which display contours of μ_t/μ_∞ for the SA and SST models at $M = 0.2$ and $Re_L = 1\times10^6$, where L is unit 1 of the grid. This is the expected typical behavior for turbulence models that are run fully turbulent (with no laminar regions imposed): turbulence initiates generally very near the leading edge. Velocity profiles in wall units at the fully turbulent locations near $x = 0.5$ and 1.5 are shown in Figs. 2a and 2b, along with results the two models would give at very large Reynolds numbers.[‡] Both models exhibit good comparisons with the law-of-the-wall theory (with the particular choice of constants $\kappa = 0.41$, $B = 5.0$) [19].

Skin friction coefficients are shown in Fig. 3, in comparison with the empirical level given by $c_f = .025Re_x^{(-1/7)}$. Downstream of the leading-edge area, both models agree well with each other and with theory. For example, at $Re_x = 1\times10^6$, both models are within about 0.3% of each other, and predict c_f about 1% low compared to theory. However, as is well known, the turbulence models actually do not activate immediately at the leading edge, but rather at a finite distance downstream of the leading edge that varies with freestream conditions.

To explore the effect of Mach number on the location of turbulence model activation, computations were run at various Mach numbers ranging from $M = 0.2$ through $M = 7$. For $M \le 2$, the Reynolds number used was $Re_L = 100,000$ per unit length of the grid (or $Re = 200,000$ over the entire plate). At higher M, it was necessary to run at higher Re_L to achieve activation on the plate.

Results showing the activation Re_x (the Re_x at which μ_t/μ_∞ first reaches 1) for $0.2 < M < 2.0$ for both models are given in Fig. 4. The figure indicates that the SA model reaches $\mu_t/\mu_\infty = 1$ near $Re_x =$ 20,000–25,000 or so across this Mach number range, whereas SST goes turbulent somewhat later near $Re_x = 40,000$ at $M = 0.2$ and near 60,000 at $M = 2.0$. Limited results using the unstructured-grid code FUN3D, given by the solid symbols, indicate similar trends as those predicted by the CFL3D code. This rules out a possible peculiarity in the latter code. Also shown in the figure are the effects of grid density, which tend to be somewhat greater for SST than for SA.

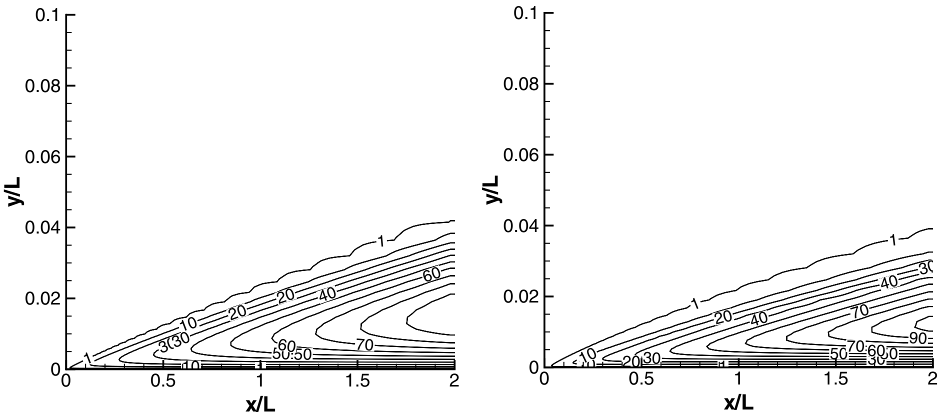
The behavior of skin friction in the "transition" region is shown in Fig. 5, in this case at $M = 0.2$ (trends at other M are similar). The SA model exhibits a very gradual transition behavior from laminar to turbulent, approaching the turbulent theory curve from below. The SST model on the other hand exhibits a more "verisimilar" rapid transition behavior from laminar to turbulent, its skin friction overshooting the theory in the early stage, which is normal because the boundary layer is thinner. Further insight can be gained by looking at u^+ vs y^+ plots shown in Figs. 6a and 6b. Both models show a very gradual approach toward turbulent log-layer behavior with increasing Re_x. Because SST yields a broader logarithmic overlap region than SA, it is difficult to compare the models directly in the region between $y^+ = 10$ and 30, but it appears that the SST model achieves self-similar behavior in this lower part of the log layer somewhat earlier. Similar trends are exhibited for $M = 2$, as shown in Figs. 7a and 7b.

We next explore differences in how the two models behave in this transitional region. Figure 8a shows a plot of peak μ_t/μ_∞ for both models in the boundary layer at $M = 0.2$, for several freestream μ_t/μ_∞ levels. For the three highest freestream turbulence levels shown here, the SA model behaves consistently. (One of these is the recommended level from Spalart and Rumsey [6] of $\tilde{\nu}'_\infty = 3$ corresponding with $\mu_t/\mu_\infty = 0.21044$.) At the lowest freestream level of $\tilde{\nu}'_\infty = 0.517301$ corresponding with $\mu_t/\mu_\infty = 0.0002$, the SA model remains laminar in this case. This laminar behavior is due to the presence of the f_{t2} term in the model, which was designed to make $\tilde{\nu} = 0$ a solution to the equations with a small basin of attraction, so that numerical tripping could be delayed for transitional flows. The SST model exhibits greater overall influence of the freestream turbulence levels. Here, the middle SST curve has been generated using the recommended levels [6].

Figure 8a shows one aspect of the different behaviors of the models, but it fails to explain why SA does not exhibit the same type of laminar-to-turbulent transition as SST. Figure 8b shows eddy viscosity profiles at three streamwise stations where each model achieved a peak of approximately $\mu_t/\mu_\infty = 0.2$, 1.0, and 8.0, respectively. In the region near the wall (below the peak), SA produces consistent levels even when peak μ_t/μ_∞ is very low. With these consistent levels, SA does not have low enough eddy viscosity to behave laminar; hence its c_f departs from laminar behavior quite early. SST, on the other hand, exhibits very different behavior. For the location where peak $\mu_t/\mu_\infty = 0.2$, its near wall eddy viscosity is much lower than it is at the downstream stations. Thus, SST behaves laminar upstream and produces a well-defined region of transition to turbulence.

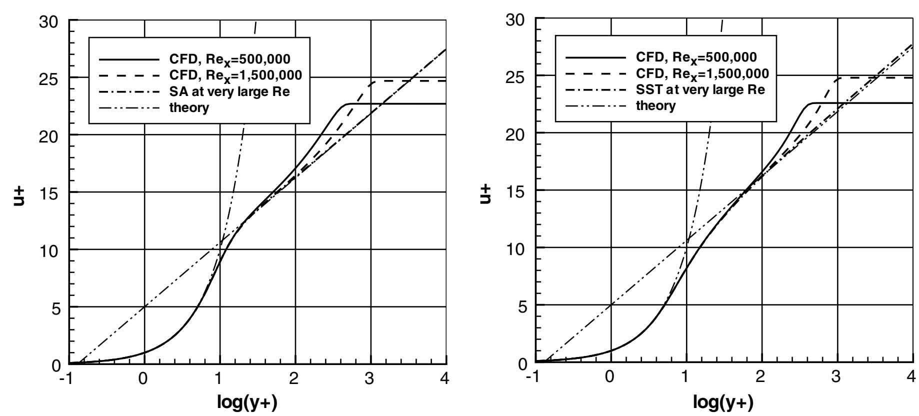
In summary, the SA model exhibits the start of turbulence activation earlier than SST (in the sense that μ_t/μ_∞ reaches 1 within the boundary layer at a lower Re_x). Furthermore, SA's behavior near the wall causes its skin friction to appear transitional over a greater distance, whereas SST displays a well-defined laminar-to-turbulent trip behavior. In terms of log-law behavior, both models gradually approach the correct slope, with SA perhaps taking slightly longer. However, these differences are probably not too important. The main point here is that for Mach numbers less than 2, both models behave laminar at very low $Re_x <$ 20,000–60,000 and then do not truly exhibit what might be considered turbulent behavior until $Re_x >$ 100,000–300,000 or so.

[‡]Strelets, personal communication, 2006.



a) SA model b) SST model

Fig. 1 Contours of μ_t/μ_∞ for subsonic flow over flat plate, $M = 0.2$, $Re_L = 1 \times 10^6$, medium grid.



a) SA model b) SST model

Fig. 2 Velocity profiles in wall units for subsonic flow over flat plate, $M = 0.2$, $Re_L = 1 \times 10^6$, medium grid.

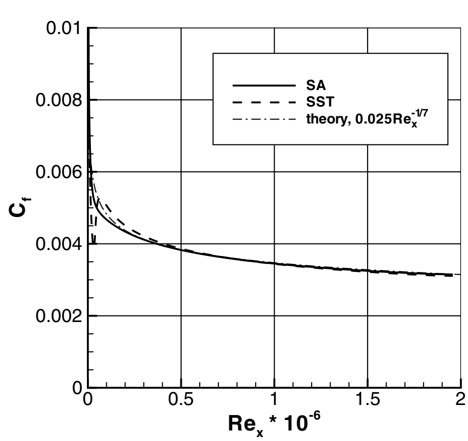


Fig. 3 Skin friction coefficients for subsonic flow over flat plate, $M = 0.2$, $Re_L = 1 \times 10^6$, medium grid.

With the new ability to sustain freestream levels in the SST model as described in Sec. III, we can now investigate the effect of freestream levels of turbulence on the transitional behavior of the SST model with better transparency (without dependency on the grid size in the far field). It is also possible to adjust the freestream level of $\tilde{\nu}$ in the SA model, corresponding to freestream μ_t/μ_∞. Results for the flat plate at $M = 0.2$ are shown in Figs. 9a and 9b. For the SA model, five values of freestream μ_t/μ_∞ were chosen. For SST, five values of freestream turbulence intensity (in percent) $\mathrm{Tu} = 100\sqrt{(2/3)k/u_\infty^2}$ were chosen. In all SST cases, $\omega L/u_\infty$ was held fixed at 5, so μ_t/μ_∞ varied as shown in the figure legend.

For SA, freestream $\tilde{\nu}'_\infty = 3$ corresponding with $\mu_t/\mu_\infty = 0.21044$ is the one recommended by Spalart and Rumsey [6]. The freestream value $k/u_\infty^2 = 1 \times 10^{-6}$, corresponding to Tu$=$ 0.08165%, is the one recommended for SST. The SA model can be steered to yield laminar flow with very low values of freestream turbulence; in this example both $\mu_t/\mu_\infty = 0.0002$ and $\mu_t/\mu_\infty = 2 \times 10^{-6}$ yielded laminar flow. The SA model shows little difference

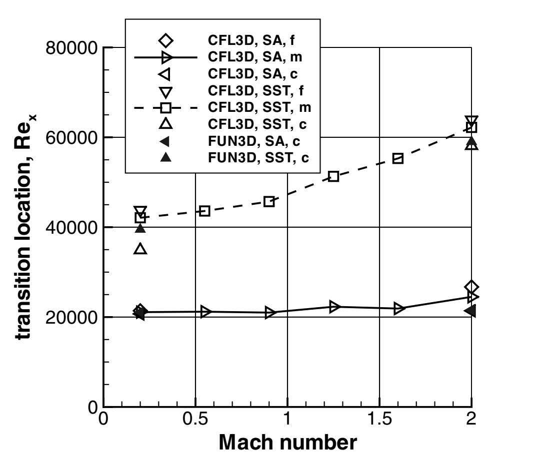


Fig. 4 Flat plate Re_x location where models go turbulent (defined by $\mu_t/\mu_\infty \geq 1$), including effect of grid density (c = coarse grid, m = medium grid, and f = fine grid).

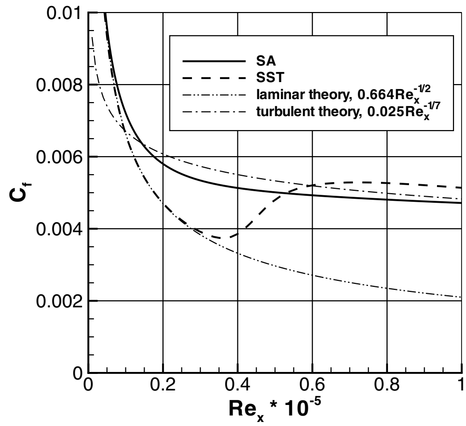


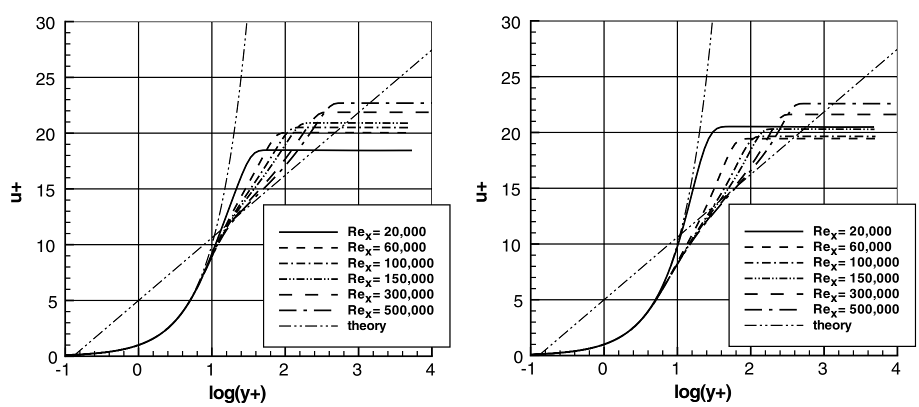
Fig. 5 Transitional behavior of the models: skin friction coefficient on medium grid at $M = 0.2$.

for the next two higher freestream turbulence levels. As discussed earlier, because of the more gradual way that SA approaches fully turbulent behavior, it is difficult to designate a location where turbulence is actually achieved. For $\mu_t/\mu_\infty = 20$, the c_f is noticeably higher everywhere. For SST, the correct overall trend for transition to turbulence is exhibited: the higher the freestream Tu, the further forward the transition, although again at the highest Tu the downstream c_f levels are noticeably higher.

It should be stressed that the transition behavior shown in Fig. 9b may or may not correspond quantitatively with the experimentally measured behavior. The SST model was certainly not designed to do so, and we are not trying to establish validity for transition predictions. However, it is reassuring to note that SST does exhibit the correct trend. This, then, seems to offer the user some level of control for achieving reduced regions of laminar flow when running at low Reynolds numbers, not that it could be extended with confidence to cases with pressure gradient. As discussed by Spalart and Rumsey, however, there are practical limits on freestream k/u_∞^2. If set too high (say, Tu > 1%), then the boundary-layer levels may be influenced through diffusion, or the sustaining terms may become high compared to destruction terms in the boundary layer and affect the turbulence budget near the wall. This is likely the reason why the downstream skin friction for $\mu_t/\mu_\infty = 20$ is higher than the other three. Hence, there are practical limits to the ability to achieve turbulent flow at low Reynolds numbers with these models.

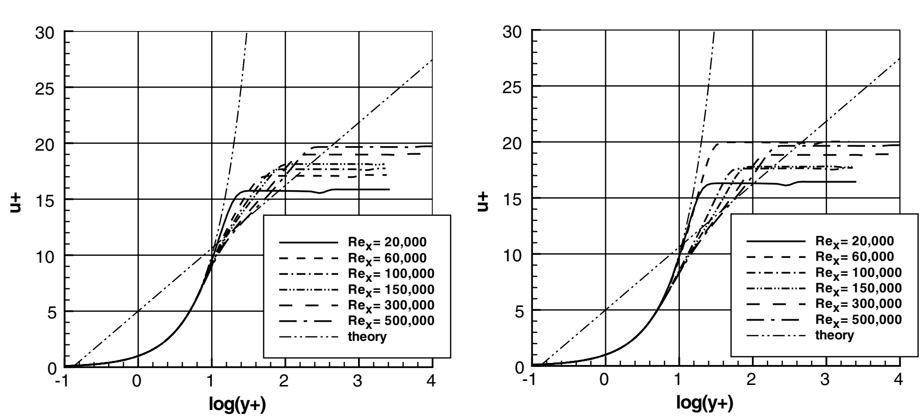
At Mach numbers higher than $M = 2$, both models have a greater tendency to remain laminar, with the SA model particularly reluctant to activate turbulence above approximately $M > 5$, as shown in Fig. 10. For example, when using the recommended level of $\tilde{\nu}'_\infty = 3$ (corresponding with $\mu_t/\mu_\infty = 0.21044$) at $M = 6.4$, the SA model does not reach $\mu_t/\mu_\infty = 1$ until near $Re_x = 1,172,000$. The SST model goes turbulent at this Mach number near $Re_x = 400,000$. However, as noted above, by increasing $\tilde{\nu}'_\infty$ in the SA model, turbulence can be triggered earlier. An example is shown in the figure for $M = 6.4$ using $\tilde{\nu}'_\infty = 4.480729$ (corresponding with $\mu_t/\mu_\infty = 0.9$). In this case the SA model activates turbulence near $Re_x = 400,000$.

Thus, for hypersonic Mach numbers, it may be more difficult to activate turbulence with these models, requiring either running at higher Reynolds numbers or employing higher freestream turbulence levels. Note that the current forms of the turbulence models used here do not attempt to account for specific compressibility



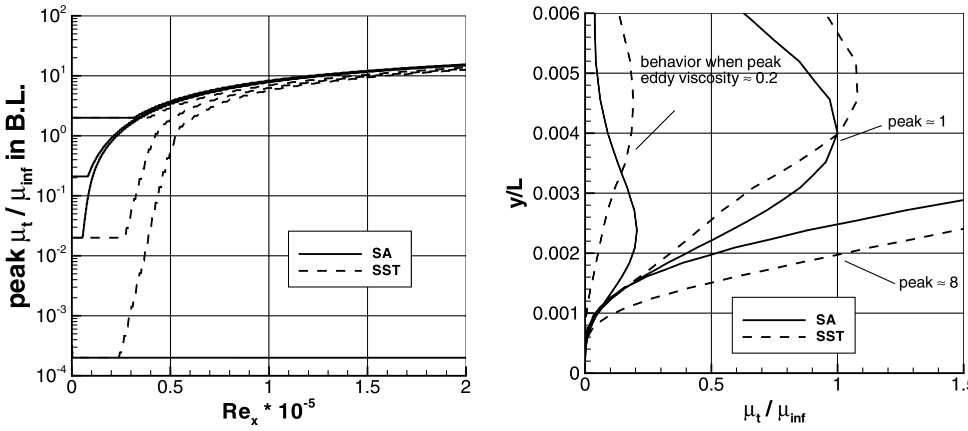
a) SA model **b) SST model**

Fig. 6 Velocity profiles in wall units in transitional region over flat plate, $M = 0.2$, medium grid.



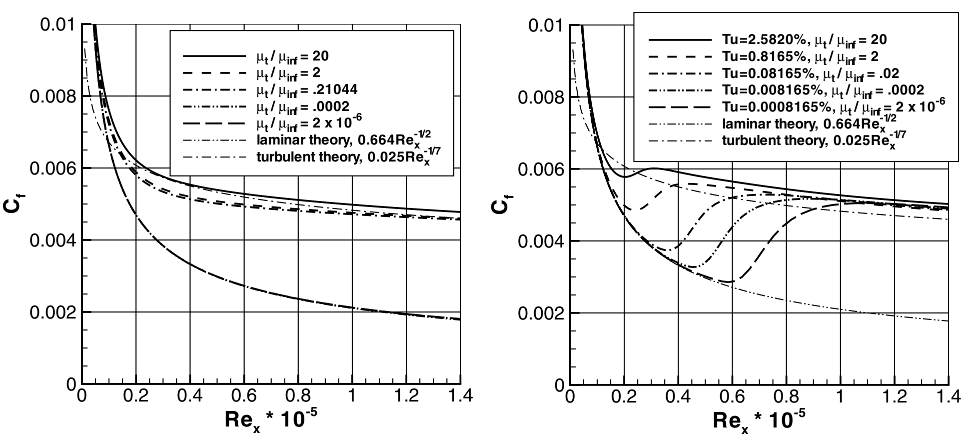
a) SA model **b) SST model**

Fig. 7 Velocity profiles in wall units in transitional region over flat plate, $M = 2.0$, medium grid.



a) b)

Fig. 8 Eddy viscosity over flat plate, $M = 0.2$, medium grid; a) peak value for various freestream turbulence levels; b) vertical profile at locations where peak $\mu_t/\mu_\infty \approx 0.2$, 1.0, and 8.0.



a) SA model b) SST model

Fig. 9 Skin friction coefficients for subsonic flow over flat plate showing effect of freestream μ_t/μ_∞ and Tu, $M = 0.2$, medium grid (dash-dotted lines correspond with recommended freestream levels from Spalart and Rumsey [6]).

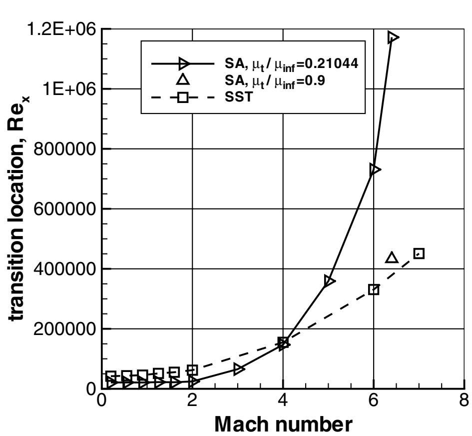


Fig. 10 *Re_x* location where models go turbulent (defined by $\mu_t/\mu_\infty \geq 1$) for flat plate at higher Mach numbers, medium grid.

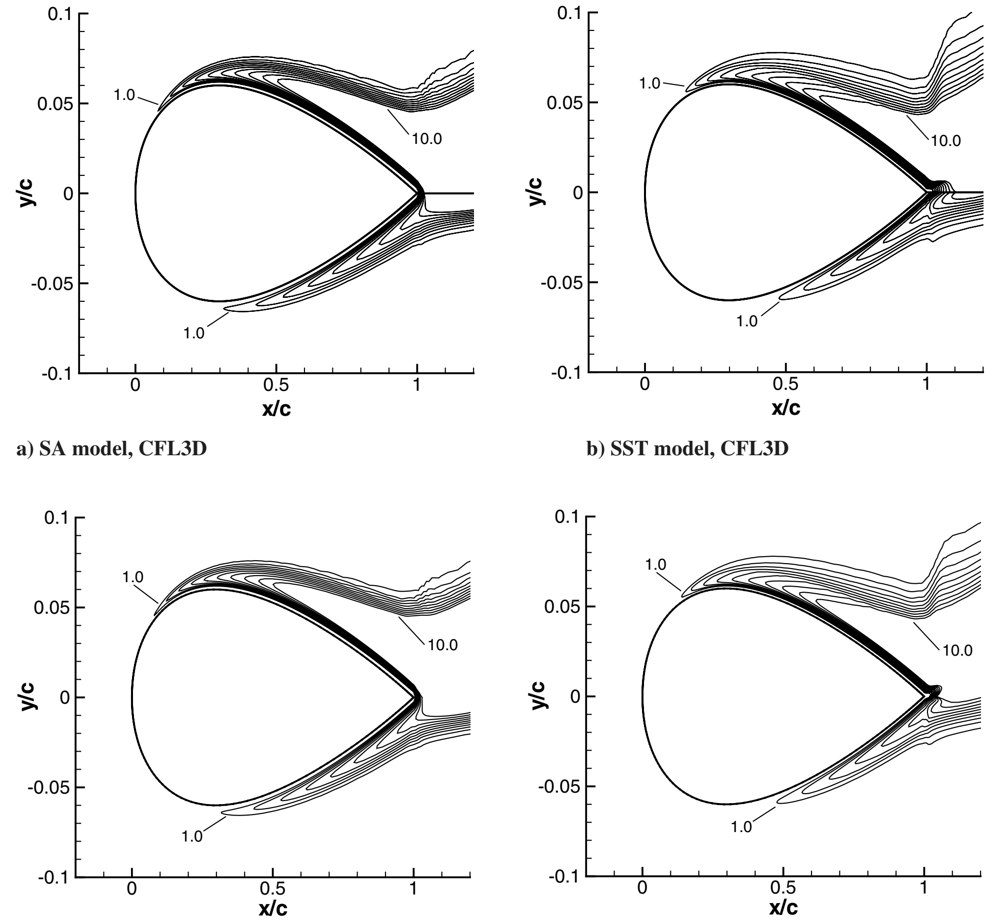
effects. However, the incompressible forms of the models (with mean density variations accounted for in the compressible Navier–Stokes equations) are expected to be reasonably accurate over a wide range of supersonic freestream Mach numbers [20], depending in part on the magnitude of the local parameter $M_t = \sqrt{2k}/c$, where k is the turbulent kinetic energy and c is the speed of sound. There are compressible forms and/or specific corrections for these models (see, for example, Catris and Aupoix [21] and Wilcox [22]), but these were not tested here.

B. NACA 0012

A second set of test cases was run for the NACA 0012 airfoil, using a family of C grids. The finest grid had 513×257 points, with 353 points on the airfoil surface, nondimensional chord length of $c = 1.0$, and minimum normal spacing at the wall of 6×10^{-7}. The far field was located at $50c$. The medium grid (for which most runs were made) used every other point (257×129) with minimum normal spacing at the wall of 1.2×10^{-6}, and the coarse grid (129×65) was every other point of this. These minimum spacings yielded average minimum y^+ levels at the wall well less than 0.1 at all conditions tested.

Based on the earlier flat plate results, it is expected that computing flow over an airfoil at low Reynolds numbers of order 100,000 will yield extensive regions of laminar flow over the forward part of the body. However, now the transition of the turbulence models will also be influenced by streamwise adverse and favorable pressure gradients. Furthermore, because the far-field boundary is so far away in most external aerodynamic cases, it is expected that the capability to maintain freestream turbulence levels without decay in the SST model will be helpful in preserving consistency as the grid is refined.

The conditions run were $Re_c = 100,000$ and $\alpha = 5$ deg, over a range of Mach numbers. Example nondimensional eddy viscosity



a) SA model, CFL3D

b) SST model, CFL3D

c) SA model, FUN3D

d) SST model, FUN3D

Fig. 11 Contours of μ_t/μ_∞ for flow over NACA 0012 airfoil, $M = 0.2$, $\alpha = 5$ deg, $Re_c = 100,000$, medium grid.

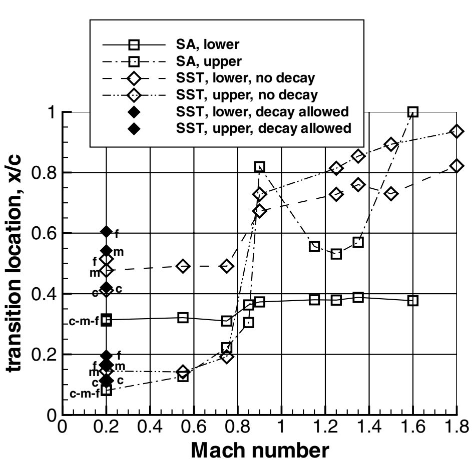


Fig. 12 NACA 0012 airfoil x/c location where models go turbulent (defined by $\mu_t/\mu_\infty \geq 1$) for $Re_c = 100,000$, $\alpha = 5$ deg, including effect of grid density at $M = 0.2$.

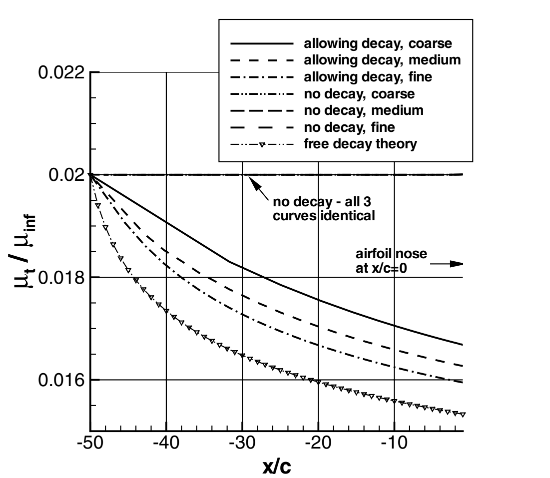


Fig. 13 Behavior of SST freestream μ_t/μ_∞ over the 50c from inflow boundary to the vicinity of the airfoil as a function of grid density, $M = 0.2$.

contours for the two turbulence models using two different codes are shown in Figs. 11a–11d for $M = 0.2$. Results for CFL3D and FUN3D are essentially the same, indicating that the results are functions of the models themselves, and not of the numerical implementation. Values of μ_t/μ_∞ first exceed 1 at $x/c = 0.081$ (upper surface) and $x/c = 0.314$ (lower surface) for SA, and further aft at $x/c = 0.145$ (upper surface) and $x/c = 0.477$ (lower surface) for SST. Thus, as much as one-third to one-half of the lower surface in these cases is laminar. In these figures, the shape of the NACA 0012 airfoil has been distorted to make it easier to visualize the extent of the turbulent regions.

As discussed earlier, we do not expect results with SA and SST to accurately represent the transitional flow physics present in low Reynolds number flowfields. This is confirmed by comparison with results from a physics-based transition code LASTRAC [23], which uses the N-factor method based on linear stability theory. For the particular case described above, the lower surface of the NACA 0012 only has an N factor of 5 as far back as $x/c = 0.77$, after which the boundary-layer code separates. Thus, the airfoil lower surface is likely mostly laminar at these conditions. The upper surface is more difficult to characterize, because the boundary-layer code used in conjunction with LASTRAC separates quickly (near $x/c = 0.06$) due to a strong adverse pressure gradient that starts near the leading edge at $x/c = 0.012$. Thus, the airfoil upper surface at this low Reynolds number is likely characterized in reality by a laminar bubble, followed by transition and turbulent reattachment [24,25]. As computed, the SA and SST models do not predict laminar separation for this airfoil at these conditions. Other experimental and theoretical published results, although at higher Reynolds numbers that may avoid laminar separation, support the LASTRAC results. Jaffe et al. [26] showed theoretical and experimental transition location for the NACA 0012 at $\alpha = 0$ deg to occur as far back as $x/c = 0.45$–0.5 at $Re = 2.5 \times 10^6$. Johansen and Sorensen [27] showed specific transition locations at $\alpha = 5$ deg to be near $x/c = 0.06$–0.13 on the upper surface and near $x/c = 0.79$–0.88 on the lower surface at $Re = 3 \times 10^6$. Ekaterinaris et al. [28] used RANS in conjunction with an empirical turbulence intermittency function for the NACA 0012 at $Re = 0.54 \times 10^6$, and found the intermittency function to go from 0 to 1 between $x/c = 0.03$ and 0.1 on the upper surface at $\alpha = 5$ deg. Gregory and O'Reilly [29] experimentally showed transition for the NACA 0012 near $x/c = 0.12$ on the upper surface and near $x/c = 0.9$ on the lower surface for $\alpha = 5$ deg and $Re = 1.44 \times 10^6$, in a wind tunnel.

As the freestream Mach number is increased, the turbulence activation locations on the airfoil using SA and SST tend to move farther downstream. A summary plot is shown in Fig. 12. Again, transition locations are based on the approximate locations where μ_t/μ_∞ first exceeds 1. The square symbols represent SA results, whose results go turbulent in the range of $0.3 < x/c < 0.4$ (lower surface) and $x/c = 0.1$ at low Mach numbers and $0.5 < x/c < 1$ for $M > 0.85$ (upper surface). The diamond symbols represent SST results, whose results go turbulent in the range of $0.5 < x/c < 0.75$ (lower surface) and $x/c = 0.15$ at low Mach numbers and $0.7 < x/c < 0.9$ for $M > 0.85$ (upper surface). Much of the upper surface loses turbulence at this Reynolds number between approximately $0.8 < M < 0.9$ because of a shift from predominantly adverse pressure gradient to predominantly favorable.

Grid resolution studies were conducted for all the cases at $M = 0.2$; these results are indicated on the left side of the figure. For SA, there was much less influence of grid on the location where $\mu_t/\mu_\infty = 1$ than there was for SST. Note also that when running SST and *allowing* decay of turbulence in the freestream, the dependence of turbulence activation on grid size was considerably greater, as indicated by the solid symbols. The reason for this increased dependence is the fact that turbulence decay rates in the far field (where grid cells tend to be very large) are influenced by grid size changes. Hence, the local ambient turbulence level in the vicinity of the airfoil, which affects turbulence development in the boundary layer, depends on the grid. Removing this dependence by maintaining freestream turbulence levels yields more consistent results.

The behavior of the eddy viscosity in the freestream for the SST model with and without turbulence decay is shown in Fig. 13, for $M = 0.2$. For comparison, the free decay theory is also shown for SST with $F_1 = F_2 = 0$ (which is equivalent to the k–ε model):

$$\mu_t = \mu_{t,\infty}\left[1 + \beta\omega_\infty \frac{x}{u_\infty}\right]^{1-\frac{0.09}{\beta}} \tag{6}$$

where $\beta = 0.0828$ and $\omega_\infty L/u_\infty = 5$. As discussed by Spalart and Rumsey [6], real flow over external aerodynamic configurations has no reason to obey the decay equations used to calibrate two-equation models in isotropic turbulence. In reality, the kinetic energy (and eddy viscosity) relevant to the aircraft flow varies very little over the size of the typical computational fluid dynamics (CFD) domain. Thus, the behavior represented by the nondecaying freestream turbulence is actually more representative of reality than the decaying behavior.

The bottom line of this airfoil study is that computing turbulent flow over airfoils at low Reynolds numbers can be problematic. Sometimes, experiments in wind tunnels are run at Reynolds numbers less than $Re_c = 500,000$, and tripping is used to ensure turbulent flow. It is important to realize that computing such flows using turbulence models like SA or SST in fully turbulent mode will likely not achieve the same flow behavior (SA does have the option of using the f_{t1} trip term). Rather, at low Re_c it is likely that the turbulence models will not become activated over much of the airfoil surface, and the higher the Mach number, the larger the laminar region is likely to be.

C. 2-D Sink Flow and 3-D Infinite Swept-Wing Flow

Having explored the behaviors of the SA and SST models at low Reynolds numbers from laminar-to-turbulent states, we now turn to the question of whether or not the models are capable of predicting relaminarization. This study was done by computing both 2-D sink flow as well as a 3-D infinite swept two-element wing.

Sink flow has been examined extensively both in experiments and computations. See, for example, Jones and Launder [30] and Spalart [31]. Here the sink flow was computed on a grid of size 257×97 between two converging plates. At inflow ($x/L = 0$), the plates were separated a nondimensional distance of $y/L = 3.6265$, and at outflow ($x/L = 20$) they were separated by $y/L = 0.1$ (i.e., the top plate converged toward the lower at an angle of 10 deg). Minimum spacing at the walls was $\Delta y/L = 1.8 \times 10^{-4}$ at inflow and 5.0×10^{-6} at outflow, which was small enough to yield an average minimum $y^+ < 0.5$ for all cases computed. At inflow, a turbulentlike velocity profile was specified along with appropriate approximated turbulence variable properties, with bulk $M = 0.01$ and pressure extrapolated from the interior. Density at inflow was set to its reference condition. At the outflow, pressure was set at $p/p_{\text{ref}} = 1.0$ and all other quantities were extrapolated from the interior. This procedure set up an accelerating flow that developed to a nearly self-similar state over approximately the last half of the channel, with near-constant skin friction coefficient based on edge conditions, and sustained streamwise acceleration parameter K_s:

$$K_s = \frac{\nu}{U_e^2}\frac{\mathrm{d}U_e}{\mathrm{d}x} \tag{7}$$

where U_e is the velocity at the edge of the boundary layer. Different values for K_s were achieved by specifying different Reynolds numbers (effectively varying ν). A plot of $c_{f,Ue}$ along the wall for three different K_s is given in Fig. 14. After the flowfield sets up, it is seen to achieve roughly constant $c_{f,Ue}$ for approximately $10 < x/L < 18$. Over this range, the edge velocity quadruples. The values of K_s were chosen to bracket the critical value of $K_s \approx 3 \times 10^{-6}$ at and above which relaminarization is considered likely [31], and also included a value double the critical value. The $c_{f,Ue}$

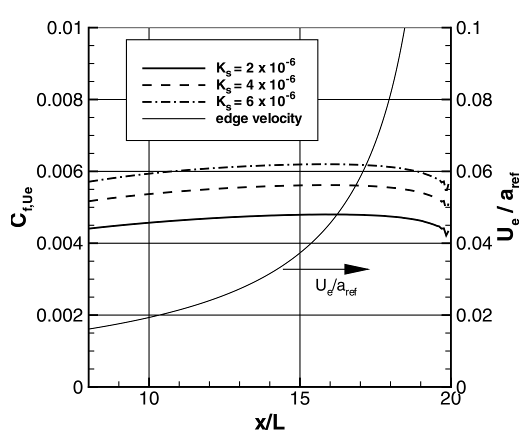


Fig. 14 Skin friction coefficients for 2-D sink flow at three different streamwise acceleration parameters using the SST model, along with computed edge velocity.

achieved for $K_s = 2 \times 10^{-6}$ in the self-similar region agrees fairly well with the turbulent value of approximately $c_{f,Ue} = 0.0046$ measured at the same K_s by Jones and Launder [30].

Figures 15a and 15b show normalized turbulent shear-stress profiles ($-\overline{u'v'}/U_e^2$) within the self-similar region for both SA and SST using three different K_s values. Both models show a slight decrease in peak $-\overline{u'v'}$ as K_s is increased, with SA decreasing slightly more than SST. The data were extracted from three different x locations in the channel to demonstrate that nearly self-similar behavior has been achieved (more so for SA than for SST). Jones and Launder [30] noted that $-\overline{u'v'}/U_e^2$ should dramatically decrease as K_s increases near these levels, and Spalart [31] showed that direct numerical simulation (DNS) computations near $K_s = 3 \times 10^{-6}$ became laminar. However, the current computations maintain turbulence even as high as $K_s = 6 \times 10^{-6}$, and do not predict the expected decrease. An example plot shows eddy viscosity for the SST model with $K_s = 6 \times 10^{-6}$ in Fig. 16; it is seen to continuously increase in the boundary layer throughout the accelerating flowfield. The SA models yield similar results. In other words, neither SA nor SST shows any evidence of relaminarization.

For the infinite swept wing, we employed the NLR 7301 wing section and used the experimental data of Viswanath et al. [32] as a guide. The NLR 7301 airfoil section (with a main element and flap) has been widely used in 2-D studies. See [33] for a summary. In [32] a wing with 45 deg sweep was built from this section, and an effort was made to approximate infinite sweep conditions by using a reasonably large span and end plates to avoid tip effects. Conditions were $Re_c = 1.3 \times 10^6$, $M \approx 0.14$, and the model angle of attack was varied from 0 to 18 deg.

The CFD was carried out on two grids with far-field extent of approximately $50c$, and spanwise extent of $0.1c$ (with 45 deg sweep included), and periodic boundary conditions (no y dependence). The fine grid had 2.74 million cells with minimum spacing at walls of $3 \times 10^{-6}c$ and 16 spanwise cells. There were 481 chordwise planar grid points on the main element surface and 449 on the flap surface. The coarse grid used every other point from the fine grid. For both grids the average minimum y^+ at the body was less than 1. A 2-D plane of the multizone fine grid is shown in Fig. 17.

In the experiment, surface flow visualization revealed a laminar separation bubble on the upper surface near the nose of the main element for all angles of attack $\alpha > 3$ deg, even though the attachment line was turbulent. Computations were performed here for $\alpha = 6$ deg. At much higher angles of attack, significant trailing-edge flow separation occurred, inhibiting convergence. Typical surface stream traces from the computations are shown in Fig. 18. Flow is from left to right. The flow attachment line (not seen) is on the lower surface of the main element near $x/c = 0.05$, and flow turning near the main element trailing edge can be seen. At these conditions the flow separates near the upper surface trailing edge of the flap.

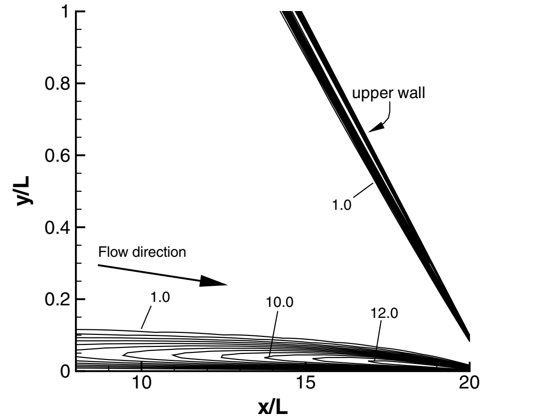


Fig. 16 Contours of μ_t/μ_∞ for sink flow, SST model, $K_s = 6 \times 10^{-6}$ (flow is from left to right, and view is expanded in the y direction for clarity).

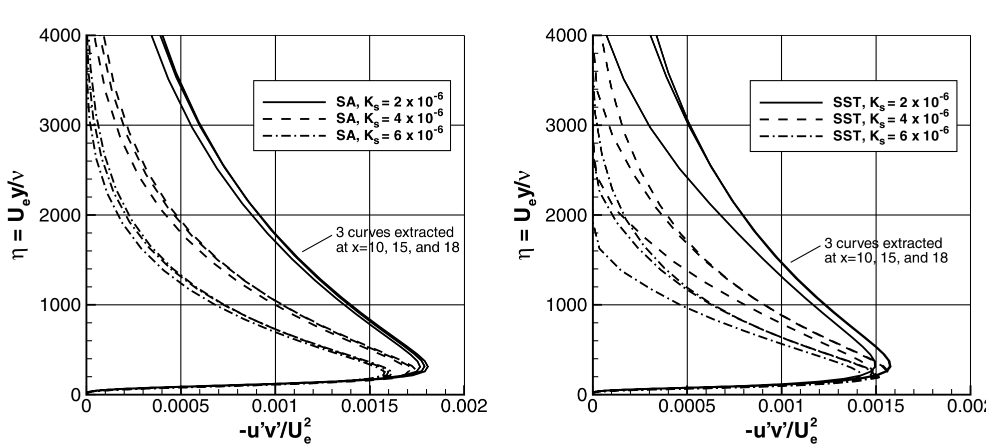
The attachment line Reynolds number $\bar{R} = Q_\infty \sin\Lambda(\eta^*/\nu)$ for these conditions in the experiment is between 200 and 250. Here, Q_∞ is the freestream total velocity, Λ is the sweep angle, and $\eta^* = \sqrt{\nu/(\partial U_e/\partial n)}$, where U_e is the inviscid edge velocity component in the $+n$ direction and n is the direction (along the surface) normal to the wing leading edge in the downstream direction. This level declared by them is lower than the critical $\bar{R} = 250$ above which the attachment line is known to sustain turbulence in experiments, flight [34–36], and DNS [37]. However, our own computation from the CFD result yielded $\bar{R} = 305$. Its calculation is rather sensitive, and our attachment line has migrated to a region with weak surface curvature. Both turbulence models produced peak eddy viscosity levels in the vicinity of the leading edge that were slightly greater than 1, and the flow remained turbulent (although weakly) everywhere downstream on the main element. Even though the skin friction is not greatly affected locally, the eddy viscosity grows again after the pressure gradient reverses from favorable to adverse, thus preventing laminar separation.

A parameter proposed by Viswanath et al. [32] to determine if relaminarization is likely in this 3-D flow is a generalization of the local streamwise acceleration parameter, K_s [Eq. (7)]:

$$K_s = \frac{\nu}{Q_e^2}\frac{\partial U_e}{\partial n}\cos^2\psi_e \tag{8}$$

where Q_e is the local edge total velocity and ψ_e is the angle between the local inviscid streamline and the $+n$ direction. We believe a more logical generalization from 2-D to 3-D would be

$$K_s = \frac{\nu}{Q_e^2}\frac{\partial Q_e}{\partial n}\cos\psi_e \tag{9}$$



a) SA model **b) SST model**

Fig. 15 Turbulent shear-stress profiles for SA and SST for 2-D sink flow at three different streamwise acceleration parameters.

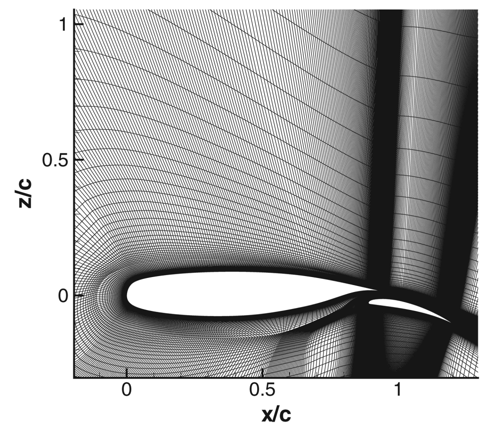


Fig. 17 2-D plane of NLR 7301 grid used for infinite swept-wing computations.

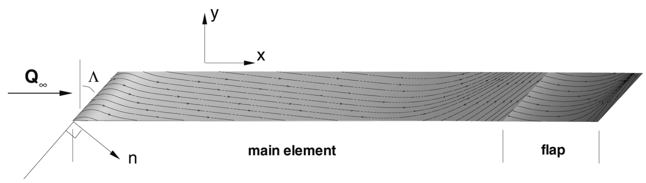


Fig. 18 Example surface stream traces for NLR 7301 infinite swept-wing computation, $Re_c = 1.3 \times 10^6$, $M = 0.14$, $\alpha = 6$ deg, and $\Lambda = 45$ deg.

but in an infinite swept flow, the two are equivalent due to the relationship $Q_e \sin \psi_e = Q_\infty \sin \Lambda = U_e \tan \psi_e$.

When K_s sustains values above 3×10^{-6}, relaminarization is likely [32,34,37]. In [32] at higher angles of attack of 15 and 17 deg, the peak K_s was nearly 1×10^{-5}. In the current computations at $\alpha = 6$ deg, the peak K_s was even higher: more than 2×10^{-5}, as shown in Fig. 19. Note that the computation of K_s can be difficult because it involves gradients of the somewhat difficult-to-determine edge value U_e (or Q_e). For low Mach numbers, one can estimate Q_e from $Q_e = Q_\infty \sqrt{1 - c_p}$ and $\cos \psi_e$ from $\cos \psi_e = \sqrt{1 - [\sin^2 \Lambda / (1 - c_p)]}$; in this case, using Eq. (9), the peak K_s only differs from that computed with the method of finding appropriate edge values (by estimating their grid index location) by about 10%. Thus, both CFD and experiment strongly indicate that relaminarization is likely in this case.

However, neither the SA nor the SST turbulence model indicates any tendency toward relaminarization. As indicated in Fig. 19, which shows results for both models on two different grids, the peak eddy viscosity in the upper surface boundary layer remains well above 1. Surface skin friction coefficients (Fig. 20) also indicate no tendency toward any sort of laminar separation bubble, which occurred near $x/c = 0.05$ in the experiment. Although not shown, we also ran cases at half the Reynolds number, for which $\bar{R}$ is appreciably less than 250. In these cases, the peak eddy viscosity in the boundary layer is lower, as expected, but it grows nonetheless and prevents laminar separation. The perfect RANS model would sustain the attachment line turbulence if $\bar{R} > 250$, even with zero ambient value (but not zero initial value), and stop sustaining as soon as $\bar{R}$ drops below 250. Here, the models incorrectly sustain turbulence even with low $\bar{R}$, just like they did with high K_s in the sink flow. Another test of SA which is not shown was to drop the freestream eddy viscosity to zero, after convergence with the usual value; it made no appreciable difference in the solution. This swept-wing behavior therefore markedly differs from the 2-D behavior in Fig. 8a.

This inability to predict laminar separation means that in practice one would need to manually "turn off" these models in the nose region in order to see any kind of bubble develop. The Baldwin–Lomax model [38] has precisely this feature of turning off eddy viscosity when the peak across the boundary layer is less than 14 times the freestream molecular viscosity, but a primary requirement in modern turbulence models is to have a local formulation, so that such a peak value is not a candidate for a modification.

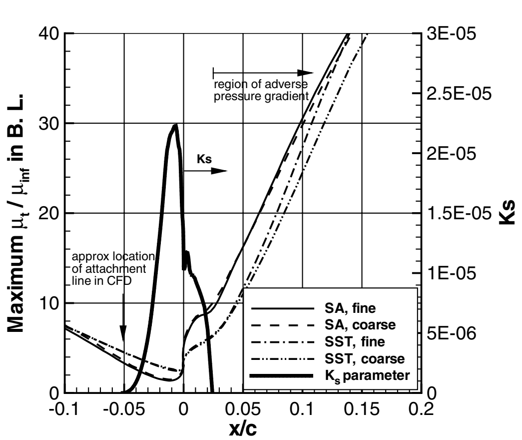


Fig. 19 Plot of K_s and peak μ_t/μ_∞ in the boundary layer for the main element of the NLR 7301 infinite swept wing, $Re_c = 1.3 \times 10^6$, $M = 0.14$, and $\alpha = 6$ deg.

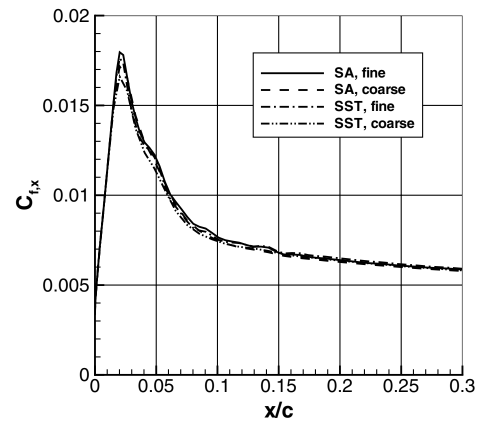


Fig. 20 Computed x-direction component of skin friction coefficient on the upper surface of the main element of the NLR 7301 infinite swept wing, $Re_c = 1.3 \times 10^6$, $M = 0.14$, and $\alpha = 6$ deg.

V. Conclusions

In conclusion, the SA and SST turbulence models, when run fully turbulent for high Reynolds number aerodynamic flows (typically on the order of $Re_L = 1 \times 10^6$ or greater, where L is the relevant geometric length scale of the body or wing), usually yield turbulent fields with only relatively small regions near stagnation points where eddy viscosity is too low to produce typical turbulent behavior. When these regions are small, they generally do not have much of an effect on the global flowfield. However, when run at lower Reynolds numbers, the models' laminar or not-fully turbulent extent can become significant compared to the geometric reference length. For zero-pressure-gradient flat plate flow, the SA model first yields $\mu_t/\mu_\infty > 1$ near $Re_x = 20{,}000$ for $0.2 < M < 2.0$. The SST model transitions somewhat later, near $Re_x = 40{,}000$ for $M = 0.2$ and near $Re_x = 60{,}000$ for $M = 2.0$. The SA model generally yields higher eddy viscosity levels in the boundary layer leading up to transition compared to SST. As a result, SA's skin friction shows more transitional behavior in the region where $\mu_t/\mu_\infty < 1$. Note that there is no absolute target behavior here, because natural transition is far from unique in reality. At higher supersonic Mach numbers, both turbulence models have a greater tendency to remain laminar, especially the SA model using the recommended freestream value of $\tilde{\nu}'_\infty = 3$. For $M > 5$ it appears to be necessary to increase its freestream $\tilde{\nu}'_\infty$ to about 5 to activate turbulence at a reasonable Re_x.

Airfoil flow has been shown to behave similarly by computing a series of examples at low $Re_c = 100,000$. Transition in these cases occurred between 10%c and the trailing edge (i.e., not at all), depending on the freestream Mach number and resulting streamwise surface pressure gradient. Again, as the Mach number is increased, airfoil flow shows a greater tendency for laminar boundary-layer behavior for both models at low Reynolds number.

The technique of maintaining freestream levels of turbulence without decay by way of "sustaining terms" in the SST model has again proved to be useful in reducing grid dependence of the model's transitional behavior. Eliminating freestream decay is also more physically realistic for external aerodynamic problems and, because it is the *local* levels of ambient turbulence that determine the turbulent behavior in the boundary layer, eliminating freestream decay makes it easier to understand and control the model's behavior. The SA model does not face this same issue; it is already designed so that its freestream level does not decay.

Flows with relaminarization are not predicted correctly with the SA or SST turbulence models. Eddy viscosities remain turbulent in accelerating or laterally strained boundary layers for which experiment and direct simulations indicate turbulence suppression. Even models more complex than the two here are usually designed with other priorities in mind than relaminarization; an exception [39] is an early version of k–ε, but the community has made little use of it over the years. The lesson to be learned from this study is that care should be exercised when using turbulence models for relaminarizing flows or flows at transitional Reynolds numbers. In these cases the turbulence models are operating outside of the range of applicability their authors have been able to address, and the flowfield physics are not likely to be predicted accurately, unless some form of transition modeling is also employed. Although strong turbulent skin friction downstream usually limits the importance of the leading-edge region, in rare instances missing the physics in this area will prevent the models from predicting a separation which would influence the entire flowfield.

Acknowledgment

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